Gerd Gigerenzer

I Think, Therefore I Err

They all laughed at Wilbur and his brother,
When they said that man could fly.
They told Marconi wireless was a phony;
It's the same old cry.

—IRA GERSHWIN

WHY DO WE MAKE ERRORS? ARE THEY BLUNDERS CAUSED BY THE limitations of our cognitive system? Or are errors indispensable parts of every intelligent system? From the first perspective, all errors are at best unnecessary and at worst harmful. Consider an error commonly made by children. When asked to find the sum of 1/2 and 1/3, the answer is often 2/5. This is called the *freshman error* of adding numerators and adding denominators (Silver, 1986). But blunders are not limited to children. After the invention of the telephone, a group of British experts concluded that this invention had no practical value, at least in their country: "The telephone may be appropriate for our American cousins, but not here, because we have an adequate supply of messenger boys" (Sherden, 1998: 175). In 1961, President John F. Kennedy is reported to have asked himself "How could I have been so stupid?" after realizing how badly he had miscalculated when he approved the Bay of Pigs invasion planned by the CIA (Janis and Mann, 1977: xv). Blunders like these seem to be unnecessary as well as embarrassing, and every intelligent system would work better without them. In this view, to err is not to think.

From the second perspective, there are errors that need to be made—that is, errors that are indispensable and functional. I call these "good" errors. Children are known for good errors. Consider a 3-year-old who uses the phrase "I gived" instead of "I gave." A child cannot know in advance which verbs are irregular; because irregular verbs are rare, the child's best bet is to assume the regular form until proved wrong. The error is "good"—that is, useful—because if the 3-year-old did not try out new forms and occasionally make errors, but instead played it safe and used only those words it had already heard, she would learn a language at a very slow rate. The characteristic of a good error is that a person is better off making the error than not making it—for reaching a goal more quickly, or attaining it at all. In this view, every intelligent system has to make errors. Making no errors would destroy the intelligence of the system. There is a close parallel to Darwinian theory, where random variability and mutation—copying errors—are essential for evolution by natural selection. Not making these errors would eliminate evolution. Trial-and-error learning, at the ontogenetic or evolutionary level, is one source of good errors in an uncertain world.

In this article, I deal with the study of human errors in experimental psychology. The problem that researchers try to resolve is this: How can one infer the laws of cognition—of perception, memory, and thought? One answer is to study the systematic errors people make. At first glance, this program looks like a straightforward extension of Francis Bacon's plan for studying nature's errors, or of Freud's strategy to analyze repressed memories, slips of tongue, and abnormal neurotic behavior. The idea is to catch nature when it does not pay attention—creating strange facts such as blood rain in Bavaria and an Irish girl with several horns growing on her body (Daston and Park, 1998). However, there is an important difference. We can easily see what is wrong with a goat with two heads or a man with obsessive-compulsive hand washing, and understand that it is not to the benefit of the animal or the human. Cognitive errors, however, are not as clear, as we will soon see. Here, one has to define rather than simply observe what an error of judgment is. In this article, I argue:

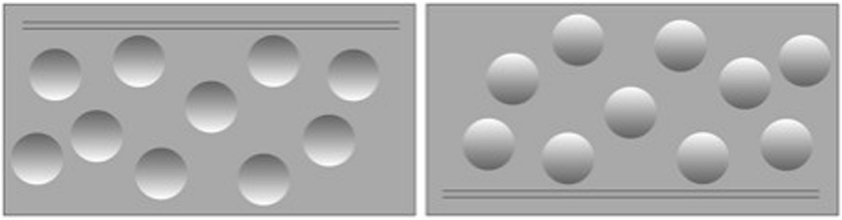

Fig. 1

1. The study of cognitive errors has been dominated by a logical definition of errors. But this narrow norm tends to mistake forms of human intelligence that go beyond logic for stupid blunders, and consequently fails to unravel the laws of mind.
2. An ecological analysis, in place of a logical one, instead reveals the existence of good errors, which open a window into the mind. The prototype of an ecological analysis is the study of visual illusions.

The method I use in this article is to document both points by illustrative examples.

VISUAL ILLUSIONS

Let us first see what a visual illusion is, and what one can learn from it. Consider the dots on the left-hand side of figure 1. They appear concave, receding into the surface away from the observer. The dots on the right side, however, appear convex: they project up from the surface, extending toward the observer. When you turn the page upside-down, the concave dots will turn into convex dots, and vice versa. But there is no third dimension, and there are no convex and concave dots. Seeing things that systematically deviate from the relevant physical measurements is called a *perceptual illusion*.

What can we learn from this illusion about how our brain works? First, that the world, from the perspective of our mind, is fundamentally uncertain. Our brain does not have sufficient information to know for certain what is out there, but it is not paralyzed by uncertainty. Second, the brain uses heuristics to make a good bet. Third, the bet is based on the structure of its environment, or what it assumes the structure to be. The brain assumes a three-dimensional world and uses the shaded parts of the dots to guess in what direction of the third dimension they extend. By experimentally varying factors such as the location of the light source and the shading, and documenting their effect on the illusion, Kleffner and Ramachandran (1992) concluded that the assumed ecological structures are that

1.) light comes from above (in relation to retinal coordinates), and
2.) there is only one source of light.

These structures describe human (and mammalian) history, where the sun and moon were the only sources of light, and only one operated at a time. The first regularity also holds approximately for artificial light today, which is typically placed above us, such as street lamps (although there are exceptions, such as car lights). The brain exploits these assumed structures by using a fast and frugal heuristic: *If the shade is in the upper part, then the dots are concave; if the shade is in the lower part, then the dots are convex.*

Shading is phylogenetically one of the most primitive cues, and so is the principle of countershading that conceals animals' shapes from predators, as in the pale bellies of swarm fishes that neutralize the effects of the sun shining from above. Helmholtz (1962 [1856-1866]) used the term "unconscious inferences" for this type of heuristic, and he and his followers (e.g., Brunswik, 1934) thought that the cues were learned from individual experience; others have favored evolutionary learning (e.g., Shepard, 1987). The systematic study of this perceptual illusion has led to various insights and speculations about the mechanism of perception. These include that for the brain, "from above" means relative to retinal coordinates, not relative to the horizon or gravity, and that our brains seem to make the "default" assumption that objects are more likely to be convex rather than concave (Deutsch and Ramachandran, 1990).

Perceptual illusions are good errors, a necessary consequence of a highly intelligent "betting" machine (Gregory, 1974). Therefore, a perceptual system that does not make any errors would not be an intelligent system. It would report only what the eye can "see." That would be both too little and too much. Too little because perception must go beyond the information given, since it has to abstract and generalize. Too much because a "veridical" system would overwhelm the mind with a vast amount of irrelevant details. Perceptual errors, therefore, are a necessary part, or by-product, of an intelligent system. They exemplify a second source of good errors: visual illusions result from "bets" that are virtually incorrigible, whereas the "bets" in trial-and-error learning are made in order to be eventually corrected. Both kinds of gambles are indispensable and complementary tools of an intelligent mind.

The case of visual illusions illustrates the general proposition that every intelligent system makes good errors; otherwise it would not be intelligent. The reason is that the outside world is uncertain, and the system has to make intelligent inferences based on assumed ecological structures. Going beyond the information given by making inferences will produce systematic errors. Not making these errors would destroy intelligence.

I. LOGIC AND BLUNDERS

Unlike in theories of perception, errors in the social sciences are typically seen as annoyances. The nuisance comes in many forms, such as observational and measurement error; statistical techniques are employed to tame the error and extract the true values from all the noise. Economics, for instance, has long tried to ignore errors of measurement, possibly because of "the absence of any good cure for this disease" (Griliches, 1974: 975). The same negative attitude toward errors has shaped the program of studying errors of judgment, which emerged in the 1960s (Edwards, 1968; Wason, 1966). It became widely known through the heuristics-and-biases program (Tversky and Kahneman, 1974), invaded social psychology (Nisbett and Ross, 1980), and shaped the emerging field of behavioral economics in the 1980s (Camerer, 1995) as well as that of behavioral law and economics in the

1990s (Sunstein, 2000). Hundreds of studies have tried to document people's blunders in almost all domains of life: flawed intuitive notions of chance, the faulty intuitions of basketball coaches, patients' illogical judgments of pain, and people's moral errors. Oddly, the new program of studying useless errors was introduced in analogy to errors in perception, specifically visual illusions.

Kahneman and Tversky argued that one can determine an error in judgment exactly like one in perception by using logic rather than physical measurement as the norm. "The presence of an error of judgment is demonstrated by comparing people's responses either with an established fact (e.g., that the two lines are equal in length) or with an accepted rule of arithmetic, logic, or statistics" (Kahneman and Tversky, 1982: 123). Just as perceptual errors help to discover the laws of perception, errors of judgment help to discover the laws of higher-order cognition, Tversky and Kahneman (1983: 313) asserted.

Psychologists were not the first to draw a parallel between perceptual and judgmental errors. In his chapter on illusion in probability estimation, Pierre Simon Laplace (1814) wrote that "the mind has its illusions, like the sense of vision" (182). Yet before the 1950s and 1960s, few psychologists thought that logic or probability theory could reveal the laws of mind. On the contrary, Wilhelm Wundt (1973 [1912]), known as the father of experimental psychology, concluded that logical norms have little to do with thought processes, and that attempts to apply them to learn about psychological processes have been absolutely fruitless.

The new focus on logic and probability was part of a larger movement. It occurred after inferential statistics was institutionalized in the social sciences during the 1950s (Gigerenzer et al., 1989), the revival of Bayesian statistics (Savage, 1954), and the emergence of theories that assumed logical structures as the basis of psychological processes (e.g., Piaget and Inhelder, 1975 [1951]).

Despite virulent disagreements with the experimental demonstrations of errors of judgment, Kahneman and Tversky's first major critic, the philosopher Jonathan Cohen (1981), also relied on the analogy with perceptual illusions and even introduced a new fashionable term, describing errors of judgment "as cognitive illusions . . . to invoke the analogy with visual illusions" (324). But what exactly does the analogy entail? An answer is strikingly absent in the literature. Recall that visual illusions are commonly understood as "good" errors, whereas errors of judgment are virtually always presented as disturbing fallacies that should have not occurred in the first place and often suggested to be the cause of many a human disaster. Given this discrepancy, the content of the analogy is less than obvious. Its function, however seems clear: the analogy served to persuade the scientific community that the laws of logic and probability were an uncontroversial norm for good thinking, just like physical measurements, and that deviations would help to unravel the laws of thought.

In what follows, I will first argue that logic failed on both goals: to define errors of judgment and to open a window into the human mind. I illustrate this argument with two logical principles, set inclusion and invariance. In the second part, I will argue that the analogy with visual illusions is actually the more promising program in reaching both goals: Kahneman, Tversky, and their followers were right in proposing the analogy, but they did not follow through on their original proposal.

Set Inclusion

In their book *The Early Growth of Logic in the Child*, Bärbel Inhelder and Jean Piaget (1964 [1959]: 101) reported an experiment in which they showed 5- to 10-year-old children pictures, of which 16 were flowers and 8 of these 16 flowers were primulas. The children were asked a list of questions about class inclusion relations, one of which was: "Are there more flowers or more primulas"? Only 47 percent of the 5- to 7-year-olds gave answers in accord with class inclusion—that is, that reflected an understanding that the flowers were more numerous because they included the primulas as a subset. Among 8-year-olds, however, a majority (82 percent) gave responses consistent with class inclusion. Later studies have confirmed this result, although some researchers suggested that the onset of class-inclusion reasoning may occur one or two years later (Reyna, 1991). Inhelder and Piaget concluded that "this kind of thinking is not peculiar to professional logicians since the children themselves apply it with confidence when they reach the operational level" (117). The facts seemed to be settled: the adolescent and adult mind is an "intuitive logician."

Without reference to this earlier work, Tversky and Kahneman (1983) reached the opposite conclusion. They referred to set inclusion problems as "conjunction problems." Consider the Linda problem:

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student she was deeply concerned with issues of discrimination and social justice, and also participated in antinuclear demonstrations.

Which of two alternatives is more probable:

Linda is a bank teller,
Linda is a bank teller and is active in the feminist movement?

The majority of undergraduates (85 percent) chose the second alternative. Tversky and Kahneman argued that this is an error of judgment, the "conjunction fallacy," because it violates logic. "Like it or not, *A* cannot be less probable than (*A and B*), and a belief to the contrary is fallacious. Our problem is to retain what is useful and valid in intuitive judgment while correcting the errors and biases to which it is prone" (Tversky and Kahneman, 1982: 98). Numerous experiments replicated this result. The facts seemed to be, once again, settled, although in the opposite direction: the adult mind is not at all an intuitive logician. The conjunction fallacy was interpreted as a potential cause of general irrationality. "[A] system of judgments that does not obey the conjunction rule cannot be expected to obey more complicated principles that presuppose this rule, such as Bayesian updating, external calibration, and the maximization of expected utility" (Tversky and Kahneman, 1983: 313). The error was afterward invoked to explain various economic and societal problems, including John Q. Public's unreasonable fear of technological risks such as nuclear reactor failures (Stich, 1985), his questionable spending on insurance (Johnson, Hershey, Meszaros, and Kunreuther, 1993), and major blunders in US security policy (Kanwisher, 1989). Stephen J. Gould (1992: 469) wrote:

> I am particularly fond of [the Linda] example, because I know that the [conjunction] is least probable, yet a little homunculus in my head continues to jump up and down, shouting at me, "but she can't just be a bank teller: read the description.". . . Why do we consistently make this simple logical error? Tversky and Kahneman argue, correctly I think, that our minds are not built (for whatever reason) to work by the rules of probability.

But why, we must ask, would 8-year-old children in Geneva not make this simple logical error, whereas American undergraduates consistently do? I argue that the irrationality is not to be found in adult reasoning but in the logical norm. Consider what the norm is: the probability of an event *A* is larger than (or equal to) the probability of the events *A* and *B*, that is, $p(A) \geq p(A \wedge B)$. This conjunction rule is used as a content-blind norm for judgment; the content of the *A*s and *B*s is not considered relevant to evaluating good reasoning. All that counts is the mathematical probability p and the logical $\wedge$ and correct judgment is attested when people use the English terms *probable* and *and* in this and only this way. This amounts to a purely syntactic definition of rational reasoning, and therefore, of an error in judgment. When a person takes account of the semantics, such as the content of *A*, or the pragmatics of the experimental situation, such as trying to find out what the experimenter wants to hear, and the resulting inference differs from the logical norm, then these forms of intelligence are counted as an error in judgment.

Are logical rules, used in a content-blind way, sound norms? I do not think so. Let us take the analogy with visual illusions seriously, specifically the aspect of uncertainty: perception cannot know the right answer, and therefore has to make an uncertain yet informed bet based on cues. One source of uncertainty in the Linda problem is the polysemy of the terms *probable* and *and*. The *Oxford English Dictionary* and its equivalents in other languages list various meanings of *probable*. A few, such as "what happens frequently," correspond to mathematical probability, but most, such as "what is plausible" and "whether there is evidence," do not. Perception solves this problem of underspecification by intelligent heuristics, and the same seems to be the case for higher-order cognition. For instance, according to Grice (1989), people rely on conversational axioms such as *relevance*. In the present context, the principle of relevance says that the description of Linda is relevant to finding the correct answer. Note that if a person treats the term *probable* as mathematical probability, then the principle of relevance is violated. You do not need to read the description of Linda to find the logical answer—and Gould's homunculus understood this point.

Consider the following version of the Linda problem, Here the polysemy of the word *probable* is eliminated by using the phrase *how many*:

There are 100 persons who fit the description above (that is, Linda's). *How many* of them are:

Bank tellers?
Bank tellers and active in the feminist movement?

This change is sufficient to make the apparently stable cognitive illusion largely disappear. In one experiment, every single participant answered that there are more bank tellers (Hertwig and Gigerenzer, 1999; for similar results see Fiedler, 1988; Tversky and Kahneman, 1983). The experiment also showed that the majority of participants interpreted *how many* in the sense of mathematical probability, but *more probable* as meaning "possible," "conceivable," or one of the other nonmathematical meanings listed in the *OED*. These results demonstrate intelligent context-sensitive reasoning (which no computer program can achieve at this point of time) rather than a rigid, content-blind application of logic (which is easy to program). The analysis also provides an answer to the question why children in Geneva made significantly fewer "errors" than American undergraduates. Inhelder and Piaget asked for *how many* rather than the ambiguous *probable,* and with this clarification, the triviality of the logical problem becomes clear and results become consistent for children and adults.

The same context-sensitivity was found for the cognitive processing of *and*. Consider this version of the conjunction:

Bank tellers and active feminists.

The conjunction has been rephrased as a noun-plus-noun phrase. This should not matter from the point of view of logical norms. However, this noun-noun phrase leads to a substantial number of violations of the conjunction rule, even when *probable* is replaced by *how many*. This result was reported by Kahneman and Tversky (1996) to defend the "reality" of the conjunction error. However, the term *and* also has no single fixed meaning, and people are equipped with intelligent heuristics to infer the intended meaning from the semantic context, not only the syntax. Specifically, noun-noun phrases often refer to the *disjunction*, not the conjunction, of two elements or classes. For instance, the announcement "We invited friends and colleagues" does not refer to the intersection between the two groups, but to the joint set of both groups. Thus, the extension of the set *friends and colleagues* is larger than that of the set *friends*, which violates the conjunction rule. But that is not an error of judgment. Consistent with this analysis, when one replaces the noun-noun phrase by *bank tellers as well as active feminists*, which largely eliminates the interpretation in terms of a disjunction, the so-called conjunction fallacy again largely disappears (Hertwig and Gigerenzer, 2004; see also Mellers, Hertwig, and Kahneman, 2001).

The moral is that human intelligence reaches far beyond narrow logical norms. In fact, the conjunction problems become trivial and devoid of intellectual challenge when people finally realize that they are intended as a content-free logical exercise. This insight was driven home to me long ago when my daughter was 8 years old, the age Inhelder and Piaget estimated that class inclusion emerges. I showed her the Primula and Linda problems that I had prepared for my students.

Q:	Are there more flowers or more primulas?
Child:	Primulas, but they all are flowers.
Q:	(Question repeated).
Child:	OK, more flowers. But why do you ask?
Q:	Is Linda more likely a bank teller or a bank teller and active in the feminist movement?
Child:	If she is in philosophy, she would not be a bank teller. Therefore, it must be bank teller and active in the feminist movement.
Q:	Why?
Child:	Because it is both. One cannot understand these questions.
Q:	Why?
Child:	Because they make no sense.

Let me summarize my argument. The use of logic and probability theory as a content-blind norm for good reasoning is widespread in recent experimental psychology. The Linda problem illustrates this norm, and how it leads to misinterpreting intelligent semantic and pragmatic inferences as mental blunders. Even children have a much more differentiated understanding of language than logic provides; they rely on conversational axioms, invited inferences, and other forms of social intelligence (Fillenbaum, 1977; Sweetser, 1990).

What have we learned from some 20 years and hundreds of experiments on the conjunction fallacy? We have learned more about the limits of logic as norms than about the workings of the mind. In fact, I do not know of any new insight that this activity has produced. Logical norms distract us from understanding intelligent behavior. Gould should have trusted his homunculus, and psychologists should trust psychological rather than logical analysis.

Framing

Framing is defined as the expression of logically equivalent information (whether numerical or verbal) in different ways. You may say that the glass is half full, or that it is half empty. A physician may tell patients that they have a 10 percent chance of dying during an operation, or a 90 percent change of surviving. In his classic *The Character of Physical Law*, Richard Feynman (1967) emphasized the importance of deriving different formulations for the same physical law, even if they are mathematically equivalent. "Psychologically they are different because they are completely unequivalent when you are trying to guess new laws" (53). Feynman used different frames in a positive way to elicit different thoughts.

In contrast to Feynman's insights, different reactions to logically equivalent formulations have been declared as normatively inappropriate, suspect of irrational thought. Consider Tversky and Kahneman's (2000 [1986]) normative principle of *invariance*: "An essential condition for a theory of choice that claims normative status is the principle of invariance: different representations of the same choice problem should yield the same preference. That is, the preference between options should be independent of their description" (211).

According to this account, it is normative to ignore whether your doctor describes the outcome of a possible operation as a 90 percent chance of survival (positive frame) or a 10 percent chance of dying (negative frame). It is logically the same (semantics and pragmatics are therefore not a topic). But patients more often accept the treatment if doctors choose a positive frame (Edwards et al., 2001). Kahneman and Tversky (2000 [1984]) interpret this to mean that people's mental machinery "is not adequate to perform the task of recoding the two versions . . . into a common abstract form" (10). From various demonstrations of framing effects, they concluded that "in their stubborn appeal, framing effects resemble perceptual illusions more than computational errors" (Kahneman and Tversky, 2000 [1984]: 5). As with violations to the conjunction rule, framing effects are seen as blunders that should not happen to a rational person.

Does invariance amount to a general definition of a judgmental error, as suggested? Feynman's insight contradicts this claim. But guessing new laws, or scientific discovery, one could argue, may be the exception to the rule. What about framing in everyday life? Consider the prototype of all framing stories:

The glass is half full.
The glass is half empty.

According to the invariance principle, (i) people's choices should not be affected by the two formulations, and (ii) if they are affected, then this violates rational choice. Should the description really not matter? Consider an experiment in which a full glass of water and an empty glass are put in front of a participant (Sher and McKenzie, 2003). The experimenter asks the participant to pour half of the full glass into the other glass, and then asks the participant to hand him the half-empty glass. Which one does the participant pick? Most people picked the previously full glass. When they were asked, however, to hand over the half-full glass, most participants picked the previously empty one. This experiment reveals that the two statements are not pragmatically equivalent (see also McKenzie and Nelson, 2003). People extract surplus information from the framing of the question, and this surplus information concerns the dynamics or history of the situation, which helps to guess what is meant. The principle of invariance is content-blind and cannot "detect" this information.

Invariance and the conjunction rule are two instances of a large number of logical principles that have been used to define errors of judgment. Others include consistency, material implication, transitivity, and additivity of probabilities, which I will not go into here. It is sufficient to say that the use of these logical rules as content-blind norms has led to the same problem: it eliminates the characteristics of human intelligence from the definition of good judgment. These include abilities that are yet unmatched by today's computer programs, such as inferring the meaning of polysemous terms from the semantic context, and decoding information that is given "between the lines." As a consequence, we have learned next to nothing about the nature of thinking or other cognitive processes from research on content-blind norms (Gigerenzer, 1996, 2001). Inappropriate norms are not simply a normative problem. They tend to suggest wrong questions, and the answers to these can generate more confusion than insight into the nature of human judgment.

II. GOOD ERRORS

Why Do We Forget?

Jorge Louis Borges tells the tale of Ireneo Funes, whom he described as having been what every man was: he looked without seeing, heard without listening, and forgot virtually everything. One day Funes was bucked off a half-tamed horse, knocked unconscious, and left crippled. But his memory became clear and without limits. He was able to recall the forms of the clouds in the sky on any day, and reconstruct every dream. He even reconstructed an entire day, although this itself took an entire day. He checked his memory against the available facts, and he found that he never made an error. It irritated him that the dog of 3:14 pm, seen in profile, should be named the same as the one seen a minute later, frontally. In Funes's world, everything was particular—which made it difficult for him to think, because to think is to forget, generalize, and abstract.

Is there a truth in Borges' story? Research on memory suggests that the answer is yes. Evolution could have produced ideal memories, and occasionally did so by mistake. The Russian psychologist A. R. Luria investigated the astounding memory of a man named Shereshevski. Luria read to him as many as 30 words, numbers, or letters, and asked him to repeat these. Whereas ordinary humans can correctly repeat about 7 plus or minus 2, this man recalled all 30. Luria increased to 50, to 70, but Shereshevski recalled all, and then repeated them in reverse order, too. Luria could not find the limits of this memory. Some 15 years after their first meeting, Luria asked Shereshevski to reproduce once again the series of words, numbers, or letters from that meeting. Shereshevski sat, his eyes closed, paused, and then first recalled the situation: that they were sitting in Luria's apartment, that Luria was wearing a gray suit sitting in a rocking chair and was reading the series to him. Then, after all those years, Shereshevski recited the series precisely from his memory. This was most remarkable at the time because Shereshevski had become a famous mnemonist who performed on stage and had been exposed to a massive amount of information to memorize in each performance, which should have buried his old memories.

Is there a cost to such unlimited memory? Shereshevski had detailed memories of virtually everything that had happened to him, both the important and the trivial. He could alter his pulse rate from some 70 to 100 by vividly remembering running after a train that had just begun to pull out. There was only one thing his brilliant memory failed to do. It could not forget. It was flooded by the images of childhood, which could cause him acute malaise and chagrin. With a memory that was composed entirely of details, he was unable to think on an abstract level. When he read a story, he could recite it word for word, but when asked to summarize the gist of the same story, he faltered.

In general, when a task required going beyond the information given, such as understanding metaphors, poems, synonyms, and homonyms, Shereshevski was more or less lost. He complained about having a poor memory for faces. "People's faces are constantly changing," he said; "it's the different shades of expression that confuse me and make it so hard to remember faces" (Luria, 1968: 64). Details that other people would forget occupied his mind, and made it hard to move from the flow of images and sensations to some higher level of awareness: gist, abstraction, and meaning. Similarly, autistic persons discriminate more accurately between true and false memories than the nonautistic do, and can have spectacular rote memory abilities. But they also remember the gist of these events less well (Schacter, 2001: 193).

Is perfect memory desirable, without error? The answer seems to be no. The "sins" of our memory seem to be good errors, that is, by-products ("spandrels") of a system adapted to the demands of our environments (Anderson and Schooler, 2000; Hertwig and Todd, 2003; Kareev, 2000; Schacter, 2001). In this view, forgetting prevents the sheer mass of details stored in an unlimited memory from critically slowing down and inhibiting the retrieval of the few important experiences. Too much memory would impair the mind's ability to abstract, to infer, and to learn. Moreover, the nature of memory is not simply storing and retrieving. Memory actively "makes up" memories—that is, it makes inferences and reconstructs the past from the present. This is in contrast to perception, which also makes uncertain inferences, but reconstructs the present from the past. Memory needs to be functional, not veridical. To build a system that does not forget will not result in human intelligence.

Why Can't Players Predict Where a Fly Ball Lands?

How does a baseball player catch a fly ball? It seems that the brain, at an unconscious level, somehow computes the trajectory of the ball. In *The Selfish Gene*, biologist Richard Dawkins writes:

> When a man throws a ball high in the air and catches it again, he behaves as if he had solved a set of differential equations in predicting the trajectory of the ball. He may neither know nor care what a differential equation is, but this does not affect his skill with the ball. At some subconscious level, something functionally equivalent to the mathematical calculation is going on (1989: 96).

If players, consciously or unconsciously, calculate the trajectory, then they should run straight to the point where the ball will hit the ground, and they should run as fast as they can to allow time to make final adjustments. However, experimental and observational studies have shown that experienced players do not live up to these expectations. First, players sometimes trot rather than run quickly, and some coaches tend to scold them because they think they are being lazy. Second, studies with baseball outfielders showed that they often run toward the ball in an arc rather than in a straight line (Shaffer et al., 2004). Third, when balls were shot from various angles into the field where players were standing, the players performed poorly in estimating the location where the ball would strike the ground (Babler and Dannemiller, 1993; Saxberg, 1987).

We seem to have identified three errors. These look like strange blunders, where players need to be educated to improve performance. But, as in the case of the dots illusion, what if these phenomena are not errors that need to be corrected, but rather the outcomes of an intelligent process? This raises the question whether players might use a heuristic rather than try to estimate the ball's trajectory. Similar to perceptual illusions, these "errors" can help to unravel the mental heuristic. Experimental studies have shown that experienced players actually use several heuristics (e.g., McLeod and Dienes, 1996; Shaffer et al., 2004). One of these is the *gaze heuristic*, which works in situations where a ball is already high up in the air:

> *Fixate on the ball, start running, and adjust your running speed so that the angle of gaze remains constant.*

The angle of gaze is the angle between the eye and the ball, relative to the ground. A player who uses this heuristic does not need to measure wind, air resistance, spin, or the other variables that determine a ball's trajectory. He can get away with ignoring every piece of causal information. All the relevant information is contained in one variable: the angle of gaze. Note that a player using the gaze heuristic is not able to compute the point at which the ball will land. But the heuristic carries the player to the point where the ball lands.

Now we can understand the nature of the three "errors." The gaze heuristic dictates the speed at which the player runs, and this can vary from trotting to running as fast as possible. Reduced speed is not an error in itself; rather, when players try to run at top speed, they may miss the ball. Similarly, running in a slight arc is not a blunder; it is a consequence of using strategies similar to the gaze heuristic, and can also be observed when a dog goes after a flying Frisbee—the dog runs so that the image of the disc is kept moving along a straight line (Shaffer et al., 2004). And finally, the player does not need to be able to compute where the ball lands; the heuristic solves the problem without that knowledge. The first two "errors" are indispensable to good performance: always running as fast as possible and in a straight line would instead prevent one from using an efficient heuristic. The third "error" has a different quality; it refers to a complex ability that the simple heuristic does not need for solving the problem.

Every Intelligent System Makes Errors

I have dealt, by means of examples, with a deep normative controversy in the cognitive and social sciences. Two visions are in conflict with one another. The first always takes errors as negative, as nuisances: the fewer one makes, the better. This negative view is implied by the reliance on logical principles for a general definition of rational behavior. I have argued that these "content-blind" norms fail to provide a reasonable definition of error, and are inapt tools for unraveling the laws of the mind. In the second view, alongside blunders of inattention and the like, there also exist good errors. A good error is a consequence of the adaptation of mental heuristics to the structure of environments. This ecological view is illustrated by visual illusions. Not making good errors would destroy human intelligence. What is correct or erroneous is no longer defined by a syntactic principle, but rather by the success of the heuristic in the real world. Good errors can provide insights into the workings of the mind.

Descartes coined the dictum "I think, therefore I am" as a first step in demonstrating the attainability of certain knowledge. In an uncertain world, however, thinking as well as elementary perception involve making bets and taking risks. To err is not only human but is a necessary consequence of this kind of intelligence. I hope Descartes will not mind a modification of his dictum accordingly: I think, therefore I err. Whenever I err, I know intuitively that I am.

NOTES

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