Tail Risk Premia vs. Pure Alpha

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We present extensive evidence that risk premium is strongly correlated with tail-risk skewness but very little with volatility. We introduce a new, intuitive definition of skewness and elicit a linear relation between the Sharpe ratio of various risk premium strategies (Equity, Fama-French, FX Carry, Short Vol, Bonds, Credit) and their negative skewness. We find a clear exception to this rule: trend following (and perhaps the Fama-French “High minus Low”), that has positive skewness and positive excess returns, suggesting that some strategies are not risk premia but genuine market anomalies. Based on our results, we propose an objective criterion to assess the quality of a risk-premium portfolio.

RISK PREMIUM: A PUZZLE

One of the pillars of modern finance theory is the concept of risk premium, i.e. that more risky investments should, on the long run, also be more profitable – otherwise, investors would divest, prices would fall and expected returns would rise until they become attractive again. Cogent as it may sound, this conclusion appears to be in contradiction with direct empirical observations. For example, several authors have reported an inverted relation between the volatility of a stock (or its $\beta$) and its excess return. This has been coined the “low volatility puzzle” in the literature. Contrarily to the intuition, less volatile stocks appear to yield higher returns [1, 2].

The problem, however, may reside in the very definition of risk. Classical theories identify risk with volatility $\sigma$. But investors are arguably not concerned by small fluctuations around the mean – they rather fear large negative drops of their wealth. These negative events are not captured by the r.m.s. $\sigma$ but rather contribute to the negative skewness of the returns. Therefore, an enticing idea is that risk premia are in fact compensating for holding assets which provide positive cash flows but may occasionally suffer large drops, erasing a large fraction of the accumulated gains. This idea has been suggested in various forms in the past, see e.g. [3–6].

In this note that summarizes our long paper [7], we discuss extensive evidence that risk premium is indeed strongly correlated with the tail risk skewness but very little with volatility, not only in the equity world but in many other sectors as well (bonds, currencies, options). We introduce a new, intuitive definition of skewness and elicit a possibly universal relation between the Sharpe ratio (SR) of risk premium strategies and their negative skewness. We find clear exceptions to this rule such as trend following that have both positive skewness and positive excess returns, suggesting that these strategies are not risk premia but genuine market anomalies. Based on our results, we propose an objective criterion to assess the quality of a risk premium portfolio.

RANKED P&Ls AND AN INTUITIVE DEFINITION OF SKEWNESS

We first checked that volatility per se is not the determinant of the excess returns of a strategy or of an investment. We studied in depth (see [7] for details) a set of stock indices across the world, deciles of Fama-French HML (High-minus-Low), SMB (Small-minus-Big)
and UMD (Up-minus-Down) since 1950, bond indices with different investment grades since 1997, or the carry trade over a wide set of currency pairs since 1974, and found little (or often negative) correlations between the Sharpe ratio of these portfolios and their volatility, confirming the “low volatility puzzle” alluded to above: see Table I.

However, something suggestive comes up when one plots the P&L of a portfolio or of a strategy (say long the US equity market since 1928, as in Fig. 1) in the following way. Instead of considering the returns in chronological order, we first sort these $N$ returns according to their absolute value and plot the cumulated P&L, $F(p)$, $p \in [0,1]$, as a function of the normalized rank $p = k/N$, starting from the return with the smallest amplitude ($k = 1$) and ending with the largest one ($k = N$). The result for the US equity market is the humped shape curve shown in red in Fig. 1, to be compared to the standard chronological time series (in black). We immediately see that while the small returns contribute positively to the average, the largest returns, contrarily, lead to a violent drop of the P&L. Strikingly, the 5% largest returns wipe out roughly half of the gains of the 95% small-to-moderate returns! The humped shape curve shown in Fig. 1 in fact characterizes all risk premium strategies that we have studied.

In the same graph, we show for comparison (in green) the “symmetrized” P&L $F_s(p)$ that would have been observed if the distribution of returns was exactly symmetrical around the same mean – i.e., such that the final point $F_s(p = 1)$ coincides with $F(p = 1)$ (see footnote [10]). In this case, one can show that the P&L is for large $N$ a monotonously increasing function of the normalized rank $p = k/N$. The comparison between real returns and symmetrized returns therefore reveals the strongly skewed nature of the returns and in fact suggests a new general definition of skewness, as the area between $F(p)$ and $F_s(p)$, after normalizing the returns such that their standard deviation is unity (see footnote [11]). To wit, we define the skewness $\zeta$ of a P&L as:

$$\zeta := -100 \int_0^1 dp \left[F(p) - F_s(p)\right],$$

(1)

where the arbitrary factor 100 is introduced such that the skewness is of order unity. In the case of the US market factor, we find $\zeta \approx -1.47$ for daily returns for a Sharpe ratio of 0.57, and $\zeta \approx -0.32$ for monthly returns. When analyzing stock markets world-wide, we find a positive correlation $\rho \approx 0.4$ between $-\zeta$ and the Sharpe ratio across different countries. [Note that all the results reported here are actually robust to the chosen definition of skewness.]

<table>
<thead>
<tr>
<th>Underlying</th>
<th>Vol/SR corr.</th>
<th>Skewness/SR corr</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bonds</td>
<td>-0.69</td>
<td>-0.36</td>
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<tr>
<td>Intl. IDX</td>
<td>-0.45</td>
<td>-0.38</td>
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<tr>
<td>SMB</td>
<td>-0.42</td>
<td>-0.89</td>
</tr>
<tr>
<td>UMD</td>
<td>-0.63</td>
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</tr>
<tr>
<td>FX Carry</td>
<td>+0.78</td>
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</tr>
<tr>
<td>HML</td>
<td>+0.03</td>
<td>+0.64</td>
</tr>
<tr>
<td>TREND</td>
<td>+0.23</td>
<td>+0.58</td>
</tr>
</tbody>
</table>

Table I: Correlation coefficient $\rho$ between volatility and Sharpe ratio, and between skewness and Sharpe ratio for all strategies investigated in [7], and for a 50-day trend following strategy across various futures markets. In most cases, correlation with volatility is found to be negative or zero, showing that the main determinant of risk premium must be skewness and not volatility. Trend and HML are clear outliers.

**RISK PREMIA IS SKEWNESS PREMIA**

We have therefore extended this skewness analysis to different contracts and different risk premia. The consistent picture that emerges from our empirical results (see Table 1) is that risk premium is not related to volatility but to skewness, or more precisely to the fact that the largest returns of that investment are strongly biased downwards. In order to bring forth an apparently universal relation between excess returns and skewness, we summarize all our results in a single scatter plot, Fig. 2, where we show the Sharpe ratios of different portfolios/strategies as a function of their (negative) skewness $-\zeta$. Quite remarkably, all but one appear to fall roughly on the regression line $S \approx 1/3 - \zeta^*/4$. This is our central result. The two parallel dashed lines correspond to a 2-$\sigma$ channel, computed with the errors on the SR and the skewness of the Fama-French strategies (other strategies have even larger error bars). Interestingly, we have also considered the returns a portfolio of four credit indices since 2004, and of the HFRX global hedge fund index, which provides us with daily data since 2003. Although this history is relatively short, the Sharpe ratio/skewness of both credit and of the hedge fund index fall in line with the global behavior (once fees are taken into account in the case of the HFRX). The outstanding exception is the 50-day trend following strategy on a diversified set of futures contracts since 1960 (see [8] for full details). In fact, a positive skewness for trend following could have been anticipated. This is because trend following is akin to a “long-gamma” strategy, and is thus expected to have a skewness of opposite sign to options [9]. Trend following excess returns appear to be a genuine market anomaly, probably of behavioral origin. We find it interesting that our universal plot in Fig. 2 allows one to identify trend following (and perhaps also HML, see the extended discussion in [7]) as a clear outlier.
Technically, this amounts to transforming the returns to a skewness that depends weakly on the average drift \( \mu \), more precisely on the ratio \( \mu/\sigma^2 \). For all purposes, this correction is extremely small, at least for daily returns (i.e. less than \( 10^{-3} \)). For a \( \mu \) independent definition of skewness, see \cite{2}, where various properties of \( F(p) \) and \( \xi^* \) are established, such as the relation between \( \xi^* \) and other, more standard definitions of skewness.

This work is the result of many years of research at CFM. Many colleagues must be thanked for their insights, in particular: P. Aliferis, A. Beveratos, L. Dao, B. Durin, Z. Eisler, A. Egloff, P. Jordan, L. Laloux, A. Landier, P. A. Reigner, G. Simon, D. Theesmar and S. Vial.