Tail Risk Premia vs. Pure Alpha

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We present extensive evidence that *risk premium* is strongly correlated with tail-risk skewness but very little with volatility. We introduce a new, intuitive definition of skewness and elicit a linear relation between the Sharpe ratio of various risk premium strategies (Equity, Fama-French, FX Carry, Short Vol, Bonds, Credit) and their negative skewness. We find a clear exception to this rule: trend following (and perhaps the Fama-French "High minus Low"), that has positive skewness and positive excess returns, suggesting that some strategies are not risk premia but genuine market anomalies. Based on our results, we propose an objective criterion to assess the quality of a risk-premium portfolio.

RISK PREMIUM: A PUZZLE

One of the pillars of modern finance theory is the concept of risk premium, i.e. that more risky investments should, on the long run, also be more profitable – otherwise, investors would divest, prices would fall and expected returns would rise until they become attractive again. Cogent as it may sound, this conclusion appears to be in contradiction with direct empirical observations. For example, several authors have reported an inverted relation between the volatility of a stock (or its β) and its excess return. This has been coined the "low volatility puzzle" in the literature. Contrarily to the intuition, less volatile stocks appear to yield higher returns [1, 2].

The problem, however, may reside in the very definition of risk. Classical theories identify risk with volatility σ. But investors are arguably not concerned by small fluctuations around the mean – they rather fear large negative drops of their wealth. These negative events are not captured by the r.m.s. σ but rather contribute to the negative skewness of the returns. Therefore, an enticing idea is that risk premia are in fact compensating for holding assets which provide positive cash flows but may occasionally suffer large drops, erasing a large fraction of the accumulated gains. This idea has been suggested in various forms in the past, see e.g. [3–6].

In this note that summarizes our long paper [7], we discuss extensive evidence that risk premium is indeed strongly correlated with the tail risk skewness but very little with volatility, not only in the equity world but in many other sectors as well (bonds, currencies, options). We introduce a new, intuitive definition of skewness and elicit a possibly universal relation between the Sharpe ratio (SR) of risk premium strategies and their negative skewness. We find clear exceptions to this rule such as trend following that have both positive skewness *and* positive excess returns, suggesting that these strategies are not risk premia but genuine market anomalies. Based on our results, we propose an objective criterion to assess the quality of a risk premium portfolio.

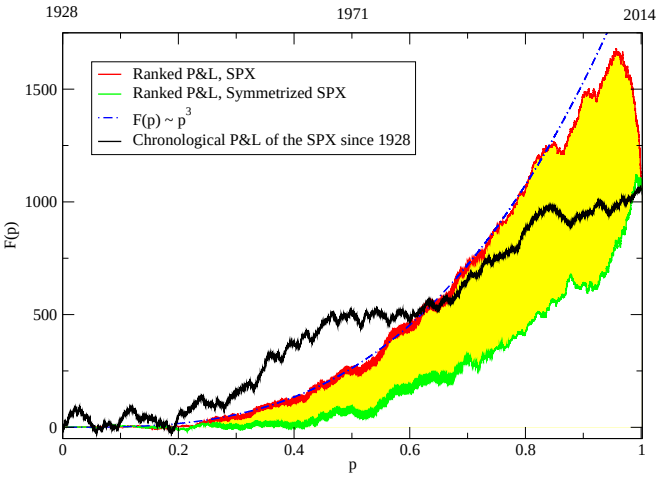

Figure 1: The ranked amplitude P&L representation: plot of the cumulated daily P&L $F(p)$ for the SPX (at constant risk) since 1928, as a function of the normalized rank of the *amplitude* of the returns, $p = k/N$ (in red). The standard, chronological P&L corresponding to holding the US equity market index at constant risk is shown in black, that by construction ends at the exact same point. We also show for comparison (in green) $F_s(p)$, corresponding to returns symmetrized around the same global mean (i.e. $F_s(p = 1) = F(1)$, again by construction). Note that while $F_s(p)$ is an increasing function of p, $F(p)$ has a humped shape and decreases abruptly when $p \to 1$, showing that returns of large amplitude contribute negatively to the P&L. Our definition of the skewness is minus the area enclosed between $F(p)$ and $F_s(p)$ (shown in yellow). One can furthermore show that $F(p)$ generically behaves as p^3 when $p \to 0$, as illustrated by the dashed line that fits the small p region of the curve well.

RANKED P&Ls AND AN INTUITIVE DEFINITION OF SKEWNESS

We first checked that volatility *per se* is not the determinant of the excess returns of a strategy or of an investment. We studied in depth (see [7] for details) a set of stock indices across the world, deciles of Fama-French HML (High-minus-Low), SMB (Small-minus-Big)

and UMD (Up-minus-Down) since 1950, bond indices with different investment grades since 1997, or the carry trade over a wide set of currency pairs since 1974, and found little (or often *negative*) correlations between the Sharpe ratio of these portfolios and their volatility, confirming the "low volatility puzzle" alluded to above: see Table I.

However, something suggestive comes up when one plots the P&L of a portfolio or of a strategy (say long the US equity market since 1928, as in Fig. 1) in the following way. Instead of considering the returns in chronological order, we first sort these N returns according to their *absolute value* and plot the cumulated P&L, $F(p), p \in [0,1]$, as a function of the normalized rank $p = k/N$, starting from the return with the smallest amplitude ($k = 1$) and ending with the largest one ($k = N$). The result for the US equity market is the humped shape curve shown in red in Fig. 1, to be compared to the standard chronological time series (in black). We immediately see that while the small returns contribute positively to the average, the largest returns, contrarily, lead to a violent drop of the P&L. Strikingly, the 5% largest returns wipe out roughly half of the gains of the 95% small-to-moderate returns! The humped shape curve shown in Fig. 1 in fact characterizes all risk premium strategies that we have studied.

In the same graph, we show for comparison (in green) the "symmetrized" P&L $F_s(p)$ that would have been observed if the distribution of returns was exactly symmetrical around the same mean – i.e. such that the final point $F_s(p = 1)$ coincides with $F(p = 1)$ (see footnote [10]). In this case, one can show that the P&L is for large N a monotonously increasing function of the normalized rank $p = k/N$. The comparison between real returns and symmetrized returns therefore reveals the *strongly skewed* nature of the returns and in fact suggests a new general definition of skewness, as the *area* between $F(p)$ and $F_s(p)$, after normalizing the returns such that their standard deviation is unity (see footnote [11]). To wit, we define the skewness ζ^* of a P&L as:

$$\zeta^* := -100 \int_0^1 \mathrm{d}p \Big[F(p) - F_s(p) \Big], \qquad (1)$$

where the arbitrary factor 100 is introduced such that the skewness is of order unity. In the case of the US market factor, we find $\zeta^* \approx -1.47$ for daily returns for a Sharpe ratio of 0.57, and $\zeta^* \approx -0.32$ for monthly returns. When analyzing stock markets world-wide, we find a positive correlation $\rho \approx 0.4$ between $-\zeta^*$ and the Sharpe ratio across different countries. [Note that all the results reported here are actually robust to the chosen definition of skewness.]

Underlying	Vol/SR corr.	Skewness/SR corr
Bonds	-0.69	-0.36
Intl. IDX	-0.45	-0.38
SMB	-0.42	-0.89
UMD	-0.63	-0.85
FX Carry	+0.78	-0.76
HML	+0.03	+0.64
TREND	+0.23	+0.58

Table I: Correlation coefficient ρ between volatility and Sharpe ratio, and between skewness and Sharpe ratio for all strategies investigated in [7], and for a 50-day trend following strategy across various futures markets. In most cases, correlation with volatility is found to be *negative* or zero, showing that the main determinant of risk premium must be skewness and not volatility. Trend and HML are clear outliers.

RISK PREMIA IS SKEWNESS PREMIA

We have therefore extended this skewness analysis to different contracts and different risk premia. The consistent picture that emerges from our empirical results (see Table 1) is that risk premium is not related to volatility but to skewness, or more precisely to the fact that the largest returns of that investment are strongly biased downwards. In order to bring forth an apparently universal relation between excess returns and skewness, we summarize all our results in a single scatter plot, Fig. 2, where we show the Sharpe ratios of different portfolios/ strategies as a function of their (negative) skewness $-\zeta^*$. Quite remarkably, *all but one* appear to fall roughly on the regression line $\mathcal{S} \approx 1/3 - \zeta^*/4$. This is our central result. The two parallel dashed lines correspond to a 2-σ channel, computed with the errors on the SR and the skewness of the Fama-French strategies (other strategies have even larger error bars). Interestingly, we have also considered the returns a portfolio of four credit indices since 2004, and of the HFRX global hedge fund index, which provides us with daily data since 2003. Although this history is relatively short, the Sharpe ratio/skewness of both credit and of the hedge fund index fall in line with the global behaviour (once fees are taken into account in the case of the HFRX). The outstanding exception is the 50-day trend following strategy on a diversified set of futures contracts since 1960 (see [8] for full details). In fact, a positive skewness for trend following could have been anticipated. This is because trend following is akin to a "long-gamma" strategy, and is thus expected to have a skewness of opposite sign to options [9]. Trend following excess returns appear to be a genuine market anomaly, probably of behavioral origin. We find it interesting that our universal plot in Fig. 2 allows one to identify trend following (and perhaps also HML, see the extended discussion in [7]) as a clear outlier.

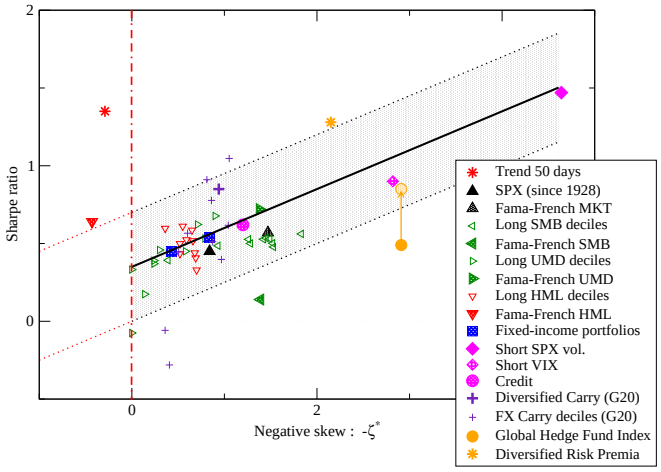

Figure 2: Sharpe ratio vs Skewness $-\zeta^*$ of all assets and/or strategies considered in [7], see legend. The relatively low Sharpe of the HFRX index can be explained by the hedge fund fees (the vertical arrow corresponds to a tentative estimate of these fees). The plain line $\mathcal{S} \approx 1/3 - \zeta^*/4$ corresponds to the regression line through all risk premia marked with filled symbols (excluding trend following). The two dotted lines correspond to a 2-σ channel, computed with the errors on the SR and on the skewness of the Fama-French strategies [7]. Note that trend following (and to a lesser extent HML) is a clear outlier marked as a red point, characterized by a positive skewness *and* a positive SR.

RISK PREMIUM VS. ALPHA

Fig. 2 not only efficiently summarizes all our results, it also suggests both a classification and a grading of different investment strategies. It is tempting to *define* a risk premium strategy as one that compensates for the skewness of the returns, in the sense that its Sharpe ratio lies within the "skewness rewarding" channel around $\mathcal{S} \approx 1/3 - \zeta^*/4$. Interestingly, the aggregate returns of hedge fund strategies fall in this channel, suggesting that a substantial fraction of hedge fund strategies are indeed risk premia.

Strategies that lie significantly below this line take too much tail risk for the amount of excess returns. On the other hand, strategies that lie significantly above this line, in particular those with *positive* skewness such as trend following, seem to get the best of both worlds. But by the same token, these strategies cannot meaningfully be classified as risk premia; rather, as argued in [8], these excess returns must represent genuine market anomalies, or "pure alpha".

From a practical point of view, our results provide an objective definition of risk premia strategies and a criterion to assess their quality based on the comparison between their Sharpe ratio and their skewness. Clearly, not all excess returns can be classified as risk premia – trend following being one glaring counter-example. On the other hand, the fact that all well known risk premia strategies seem to lie around a unique line is striking. One tentative interpretation, following e.g. [3, 5, 6] is that all these risk premia in fact reveal the same "catastrophic" risk, that would – if realized – spread out over many different asset classes. Conversely, if different risk premia are somewhat decorrelated, then skewness can be diversified away, allowing risk premia portfolios to move *above* the equilibrium line shown in Fig. 2. That this may be possible is illustrated by the orange "star" shown in Fig. 2, corresponding to a synthetic Diversified Risk Premia portfolio, with an equal weight on Long stock indices, Short Vol, FX Carry and CDS indices. We believe that this is what "good" alternative beta managers should strive to achieve. [12]

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[10] Technically, this amounts to transforming the returns r_t as: $m + \epsilon_t(r_t - m)$ where m is the average return and $\epsilon_t = \pm 1$ an independent random sign for each t.

[11] This definition leads to a skewness that depends weakly on the average drift μ, more precisely on the ratio $(\mu/\sigma)^2$. For all purposes, this correction is extremely small, at least for daily returns (i.e. less than 10^{-3}). For a μ independent definition of skewness, see [7], where various properties of $F(p)$ and ζ^* are established, such as the relation between ζ^* and other, more standard definitions of skewness.

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